2021

(July)

ECONOMICS

(Honours)

(Mathematics for Economist)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define set. Explain different operations of sets with examples. 2+3=5

(b) Given the sets

$$A = \{1, 2, 3, 4\}$$
$$B = \{2, 4, 5, 6\}$$
$$C = \{0, 3, 4, 7, 8\}$$

Prove the De Morgan's law for union and intersection. 4

(c) In a class of 25 students of economics and politics, 12 students have taken economics. Out of these 8 have taken economics but not politics. Find the number of students who have taken economics and politics and those who have taken politics but not economics. 3+3=6

2. *(a)* Differentiate any *three* of the following with suitable examples : 3×3=9

(i) Linear and quadratic functions

(ii) Homogeneous and homothetic functions

(iii) Explicit and implicit functions

(iv) Domain and range of a function

(b) Find the equation of the straight line passing through the points (3, –2) and (–4, 1). Also write down the gradient of the line. 4+2=6

UNIT—II

3. *(a)* What is matrix? Mention some of its properties. 1+5=6

(b) Solve the given simultaneous equations by matrix inversion method : 9

$$2x_1 + 3x_2 - x_3 = 15$$
$$4x_2 + 2x_3 = 16$$
$$3x_1 + 2x_2 = 18$$

4. *(a)* State the Hawkins-Simon conditions for input/output analysis. What are its implications? 3+2=5

(b) Prove that $(ABC)^T = C^T B^T A^T$. Given

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 \\ 1 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$ 4

(c) If

$$A = \begin{bmatrix} 8 & 4 \\ 3 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

find the matrix C, such that $2A + 4B - 3C = 0$, where 0 is a null matrix. 3

(d) Find the value of k, if A is a singular matrix : 3

$$A = \begin{bmatrix} 2 & 3 & 8 \\ 4 & 5 & k \\ 2 & 2 & -2 \end{bmatrix}$$

UNIT—III

5. *(a)* Evaluate (any *three*) : 3×3=9

(i) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

(ii) $\lim\limits_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$

(iii) $\lim\limits_{x \to \infty} \dfrac{5x^3 + 2}{3x^3 + x + 1}$

(iv) $\lim\limits_{x \to 2} \dfrac{x^2 - 6x + 8}{x^2 - 7x + 10}$

(v) $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x} - \sqrt{1-x}}{x}$

(b) What makes a function continuous? Given the function

$$\begin{aligned} f(x) &= 3 + 2x && \text{for } -3/2 < x \le 0 \\ &= 3 - 2x && \text{for } 0 < x < 3/2 \\ &= -3 + 2x && \text{for } x \ge 3/2 \end{aligned}$$

Is the function continuous at $x = 0$? 2+4=6

6. *(a)* Differentiate any *four* of the following functions : 2×4=8

(i) $y = (2x - 5)(x^2 + x + 1)$

(ii) $y = (2x^2 + 7)^{10}$

(iii) $y = \sqrt{x^2 + 5x}$

(iv) $y = \dfrac{x^2 + 7}{x^2 - 7}$

(v) $x^3 + 3xy + 5y - 6 = 0$

(vi) $y = x^x$

(b) Find the first- and second-order partial derivatives of the following function :

$$z = 2x^3 + 5x^2y + xy^2 + y^2$$

Verify that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$ 4

(c) $u = (3x^2 + 5y^2)^5$. Find du. 3

UNIT—IV

7. *(a)* State the necessary and sufficient conditions for maximum and minimum values, and hence find the maximum and minimum values of the function $y = \frac{1}{3}x^3 - 3x^2 + 8x + 10$. 2+4=6

(b) The demand equation for a manufacturer is given by $P = 500 - 2q$, and his average cost function is $0 \cdot 25q + 4 + \dfrac{400}{q}$, where q is output and p is price. Determine—

(i) the level of output at which profit is maximized;

(ii) the price at which this occurs;

(iii) the maximum profit. 4+3+2=9

8. *(a)* The total cost associated with producing and marketing x units of an item is given by

$$C = 0 \cdot 005x^3 - 0 \cdot 02x^2 - 30x + 3000$$

Find AC at 10 units of output and MC at 3 units of output. 3+3=6

(b) The demand function is given by $q = \dfrac{20}{p+3}$. Calculate price elasticity of demand at price $p = 2$ and also interpret the result. 7+2=9

UNIT—V

9. *(a)* What is integration? Why is there a constant of integration? 1+2=3

(b) Find the integral of the following (any *four*) : 3×4=12

(i) $\int \left(4x^3 + \dfrac{1}{\sqrt{x}} - 3\right) dx$

(ii) $\int 4(e^{2x} + x)(e^{2x} + x^2)^2 \, dx$

(iii) $\int \dfrac{8x}{(2x^2 + 1)} dx$

(iv) $\int x^2 e^x \, dx$

(v) $\int x \cdot \log x \, dx$

(vi) $\int 10^{-x} \, dx$

10. *(a)* Evaluate : 3

$$\int_2^4 3x^2(x^2 + 1) \, dx$$

(b) What is producer's surplus? If the production function is given by $Q = \sqrt{-4 + 4p}$ and the market price is 10, find the producer's surplus. 2+4=6

(c) The demand and supply functions are $P_d = (6 - q)^2$ and $P_s = 14 + q$ respectively. Find the consumer's surplus under perfect competition. 6

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